OCR
Oxford Cambridge and RSA

MODEL ANSWERS.

Wednesday 5 June 2019 – Morning

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

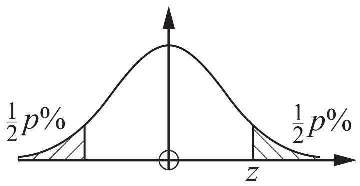
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (25 marks)

1 **In this question you must show detailed reasoning.**

Show that $\int_4^9 (2x+\sqrt{x})\,\mathrm{d}x = \frac{233}{3}$. **[3]**

① $\int_4^9 (2x+\sqrt{x})\,dx$ $x^{1/2}$

$\Rightarrow \left[\frac{2x^2}{2} + \frac{x^{1/2+1}}{1/2+1}\right]_4^9$

$\Rightarrow \left[x^2 + \frac{2x^{3/2}}{3}\right]_4^9$

$\Rightarrow \left[(9)^2 + \frac{2}{3}(9)^{3/2}\right] - \left[(4)^2 + \frac{2}{3}(4)^{3/2}\right]$

$\Rightarrow [81 + 18] - [16 + 16/3]$

$\Rightarrow 99 - \frac{64}{3}$

$= \frac{233}{3}$ as required

$x_1 \; y_1$

2 Show that the line which passes through the points $(2, -4)$ and $(-1, 5)$ does not intersect the line $3x + y = 10$. **[3]**

$x_2 \; y_2$

Finding the line that passes through the points ;

① Gradient

$$\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{5 - (-4)}{-1 - 2} \Rightarrow \frac{9}{-3} = -3$$

$$y - y_0 = m(x - x_0)$$

$$y - 5 = -3(x - (-1))$$

$$y = -3x - 3 + 5$$

$$y = -3x + 2$$

The line in the question; $y = -3x + 10$

∴ Since both of them have the same gradient, they are parallel and therefore cannot intersect.

3 The function f(x) is given by $f(x) = (1 - ax)^{-3}$, where a is a non-zero constant. In the binomial expansion of f(x), the coefficients of x and x^2 are equal.

(a) Find the value of a. **[3]**

(b) Using this value for a,

(i) state the set of values of x for which the binomial expansion is valid, **[1]**

(ii) write down the quadratic function which approximates f(x) when x is small. **[1]**

a) Expanding f(x) gives us;

$$1 + (-3)(-ax)^1 + \frac{(-3)(-4)(-ax)^2}{2!}$$

$$1 + 3ax + 6a^2x^2 \ldots$$

$$+3a = 6a^2$$

$$6a^2 - 3a = 0$$

$$3a(2a-1) = 0$$

$a = 0$ or $a = 1/2$ since $a \neq 0$ $\therefore a = \frac{1}{2}$

b) i) $(1 + 1/2 x)^{-3}$

$-1 < \frac{1}{2}x < 1 \Rightarrow -2 < x < 2$

$\therefore |x| < 2$

ii) when x is small;

$$\left(1+\tfrac{1}{2}x\right)^{-3} \Rightarrow 1+\frac{3}{2}x+\frac{3}{2}x^2.$$

4 Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end.
Determine whether a person of mass 50 kg can tip the beam by standing on it. [3]

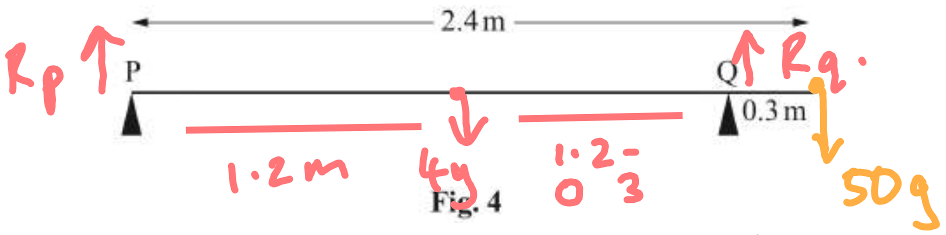


Fig. 4

Taking moments about Q:

clockwise moment = anticlockwise moment

$$50g(0.3) + R_p(2.4 - 0.3) = 4g(1.2 - 0.3)$$

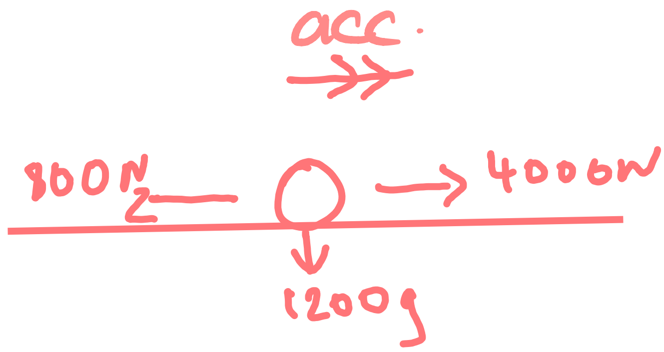
$$15g + 2.1R_p = 3.6g$$

$$11.4g = -2.1R_p$$

$$R_p = -53.2\text{N}$$

- since the answer for $R_p < 0$ the man will tip the beam.

5 A car of mass 1200 kg travels from rest along a straight horizontal road. The driving force is 4000 N and the total of all resistances to motion is 800 N.
Calculate the velocity of the car after 9 seconds. **[4]**



$F = ma$ (N2L)

$(4000 - 800) = 1200 \times a$

$$a = \frac{3200}{1200} = 8/3 \ ms^{-2}$$

s
$u = 0$
$v =$
$a = 8/3$
$t = 9$

$v = u + at$

$v = 0 + \frac{8}{3}(9)$

$v = \boxed{24 \ ms^{-1}}$

6 (a) Prove that $\frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta} = \cot\theta$. $\frac{\cos\theta}{\sin\theta}$ [4]

(b) Hence find the exact roots of the equation $\frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta} = 3\tan\theta$ in the interval $0 \leqslant \theta \leqslant \pi$. [3]

a) $\frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta}$

$$\frac{\sin\theta(\sin\theta) - (1-\cos\theta)}{\sin\theta(1-\cos\theta)}$$

$$\Rightarrow \frac{\sin^2\theta + \cos\theta - 1}{\sin\theta(1-\cos\theta)}$$ But $\sin^2\theta = 1-\cos^2\theta$

$$\frac{\cancel{1} - \cos^2\theta + \cos\theta - \cancel{1}}{\sin\theta(1-\cos\theta)} \Rightarrow \frac{\cos\theta - \cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$\Rightarrow \frac{\cos\theta\cancel{(1-\cos\theta)}}{\sin\theta\cancel{(1-\cos\theta)}} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

as required.

b) $\cot\theta = 3\tan\theta$

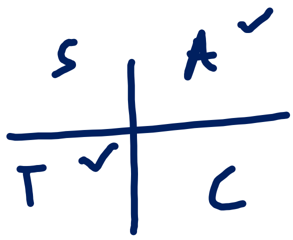
$$\cot\theta = \frac{1}{\tan\theta}$$

$$\therefore \quad \frac{1}{\tan\theta} = 3\tan\theta$$

$$\tan^2\theta = \frac{1}{3}$$

$$\tan\theta = \pm\frac{1}{\sqrt{3}}$$

<u>Option 1</u>

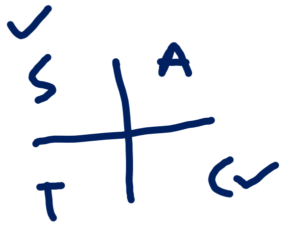


$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{6}\pi, \ \pi + \frac{1}{6}\pi$$

Only answer in the range for this option is $\pi/6$

PTO for $-\frac{1}{\sqrt{3}}$



Option 2

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\theta = \frac{5\pi}{6},\ 2\pi - \frac{\pi}{6}$$

Only answer that is in the range for this option is $\frac{5\pi}{6}$.

$\therefore$ Answers $= \frac{\pi}{6}, \frac{5\pi}{6}$

Answer **all** the questions.

Section B (75 marks)

7 The velocity $v\,\mathrm{m\,s^{-1}}$ of a particle at time t s is given by

$v = 0.5t(7-t)$.

Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$. **[4]**

expand $v = 0.5t(7-t)$

$v = 3.5t - 0.5t^2$

$\frac{dv}{dt} = 3.5 - 0.5(2)t$

$= 3.5 - t$

when $t = 8$ $\frac{dv}{dt}$ (which is also known as acceleration)

$= 3.5 - 8$

$= -4.5\,\mathrm{ms^{-2}}$

when $t = 8$ what is the velocity?

$v = 3.5(8) - 0.5(8)^2$

$v = -4\,\mathrm{ms^{-1}}$

→ since both velocity and acceleration are negative, speed is increasing.

8 An arithmetic series has first term 9300 and 10th term 3900.

(a) Show that the 20th term of the series is negative. [3]

(b) The sum of the first n terms is denoted by S. Find the greatest value of S as n varies. [4]

a) $a = 9300$

$a + d(n-1) \Rightarrow n^{th}$ term.

$9300 + d(10-1) = 3900$

$9d = 3900 - 9300$

$d = \frac{-5400}{9} = -600$

20^{th} term $= 9300 - 600(20-1)$

$= -2100$ which is negative

b) The sum will increase until the first negative term $\therefore$ we need to find the first -ve term.

$a + d(n-1) < 0$

$9300 - 600(n-1) < 0$

$600(n-1) > 9300$

$n - 1 > 15.5 \Rightarrow n > 16.5$

$\therefore$ the largest n term which is +ve is $n = 16$

$\Rightarrow$ Sum to first 16 terms should give us S

$S_n = \frac{n}{2}[2a + d(n-1)]$

$S_{16} = \frac{16}{2}[2(9300) - 600(16-1)]$

$S = 76,800$

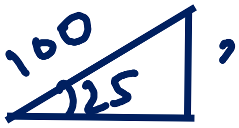
9 A cannonball is fired from a point on horizontal ground at $100\,\text{m s}^{-1}$ at an angle of 25° above the horizontal. Ignoring air resistance, calculate

(a) the greatest height the cannonball reaches, [3]

(b) the range of the cannonball. [4]

a)

Finding the vertical component of the velocity



$\sin 25 = \frac{opp}{100}$

$opp = 42.3\,\text{ms}^{-1}$

(we are taking ↑ to be +ve)

$s = ?$
$u = 42.3$
$v = 0$
$a = -9.8$
$t = x$

remember;
at max height vertical component of velocity $= 0$.

$v^2 = u^2 + 2as$

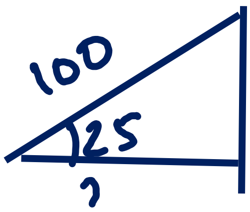
$0 = (42.3)^2 + 2(-9.8)(s)$

$19.6s = 1786.1$

$s = 911\text{m}$ (3sf)

b) Range is total horizontal distance travelled

① Horizontal component of initial velocity:



$\cos 25 = \frac{?}{100}$ $? = 90.6\text{ms}^{-1}$

② From the time taken to reach max height, doubling that should give us the total time of flight;

→ From (a)

s
$u = 42.3$
$v = 0$
$a = -9.8$
$t = ?$

$v = u + at$

$0 = 42.3 - 9.8t$

$t = \frac{42.3}{9.8} = 4.32$

- Doubling $t = 8.63$

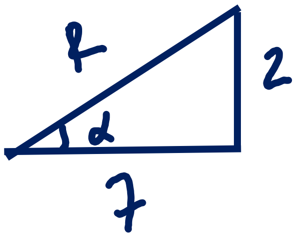
(3) Range = initial horizontal component of velocity × time

$\Rightarrow 90.6 \times 8.63 = 782\,m$ (3sf)

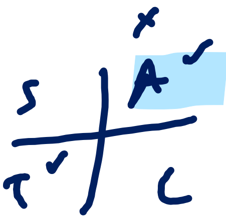
10 **(a)** Express $7\cos x - 2\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of R and the value of α correct to 3 significant figures. **[4]**

(b) Give details of a sequence of two transformations which maps the curve $y = \sec x$ onto the curve $y = \dfrac{1}{7\cos x - 2\sin x}$. **[3]**

10 a)



$R = \sqrt{(7)^2 + (2)^2} = \sqrt{53}$



$\alpha = \tan^{-1}(2/7)$ ← From diagram

$= 0\ 278$ (3sf)

$\Rightarrow \sqrt{53}\cos(x + 0\ 278)$

b) $= \dfrac{1}{7\cos x - 2\sin x} = \dfrac{1}{\sqrt{53}\cos(x + 0.278)}$

$= \dfrac{1}{\sqrt{53}}\sec(x + 0.278)$

$\therefore$ 1 Stretch scale factor $\frac{1}{\sqrt{53}}$

2 Translation $\begin{pmatrix} -0.278 \\ 0 \end{pmatrix}$

11 In this question, the unit vector **i** is horizontal and the unit vector **j** is vertically upwards.

A particle of mass 0.8 kg moves under the action of its weight and two forces given by $(k\mathbf{i}+5\mathbf{j})$ N and $(4\mathbf{i}+3\mathbf{j})$ N. The acceleration of the particle is vertically upwards.

(a) Write down the value of k. **[1]**

Initially the velocity of the particle is $(4\mathbf{i}+7\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(b) Find the velocity of the particle 10 seconds later. **[4]**

a) Since the acceleration is vertically upwards → i component of resultant force = 0

$$\therefore \begin{pmatrix} k \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} k+4 \\ 8 \end{pmatrix}$$

$$k+4=0$$

$$k=-4$$

b) Since acceleration is only vertically upwards, only consider vertical motion

$$F - \text{Weight} = ma$$

$$5+3-0.8g = 0.8a$$

$= 0.16 = 0.8a$

$a = 0.2 ms^{-2}$

s
$u = 7$
$v = ?$
$a = 0.2$
$t = 10$

$v = u + at$

$v = 7 + 0.2(10)$

$= 9$

∴ since horizontal motion remains constant, (4)

$\text{velocity} = \begin{pmatrix} 4 \\ 9 \end{pmatrix} = 4i + 9j$

y = 8t not 4t

Question was wrong.

12 Fig. 12 shows a curve C with parametric equations $x = 4t^2$, $y = 4t$. The point P, with parameter t, is a general point on the curve. Q is the point on the line $x + 4 = 0$ such that PQ is parallel to the x-axis. R is the point (4, 0).

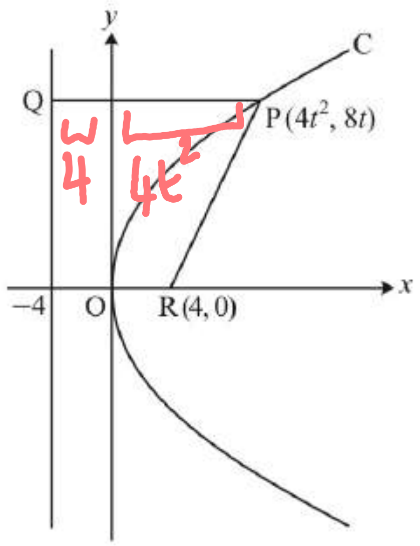


Fig. 12

(a) Show algebraically that P is equidistant from Q and R. **[4]**

(b) Find a cartesian equation of C. **[2]**

a) $x = 4t^2$ $y = 8t$

From the diagram,

$Q \rightarrow (-4, 8t)$

∴ distance of $PQ \rightarrow 4 + 4t^2$

distance of PR.

$(4, 0)$ $(4t^2, 8t)$

$$\Rightarrow PR^2 = (8t)^2 + (4t^2 - 4)^2$$

$$= 64t^2 + 16t^4 - 32t^2 + 16.$$

$$= \underbrace{16t^4 + 32t^2 + 16}_{\text{this is } = (4t^2+4)^2}.$$

this is PQ

$$\therefore PR^2 = (PQ)^2$$

$\therefore PR = PQ$

$\therefore$ they are equidistant as required

b) $x = 4t^2$

$y = 8t \longrightarrow t = \dfrac{y}{8}$ – ①

Replacing ① into x

$$x = 4\left(\frac{y}{8}\right)^2 \Rightarrow x = \frac{y^2}{16} \quad \therefore y^2 = 16x$$

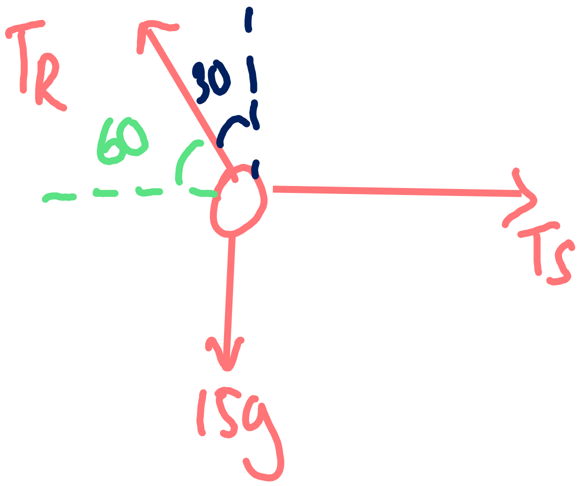
13 A 15 kg box is suspended in the air by a rope which makes an angle of 30° with the vertical. The box is held in place by a string which is horizontal.

(a) Draw a diagram showing the forces acting on the box. [1]

(b) Calculate the tension in the rope. [2]

(c) Calculate the tension in the string. [2]

a)



b) Resolving forces,

$T_R \cos 30 = 15g$

$T_R = \dfrac{15g}{\cos 30} = 98\sqrt{3} = 170\,N$ (3sf)

c) $T_R \cos 60 = T_S$

$98\sqrt{3} \cos 60 = 49\sqrt{3} = 84.9\,N$

14 Fig. 14 shows a circle with centre O and radius r cm. The chord AB is such that angle AOB $= x$ radians. The area of the shaded segment formed by AB is 5% of the area of the circle.

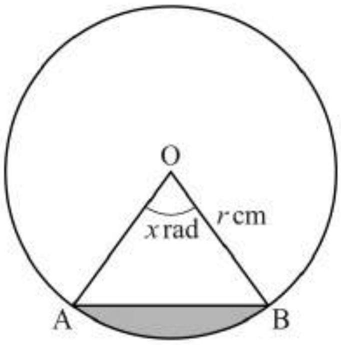


Fig. 14

(a) Show that $x - \sin x - \frac{1}{10}\pi = 0$. **[4]**

The Newton-Raphson method is to be used to find x.

(b) Write down the iterative formula to be used for the equation in part **(a)**. **[1]**

(c) Use three iterations of the Newton-Raphson method with $x_0 = 1.2$ to find the value of x to a suitable degree of accuracy. **[3]**

a) Area of sector — ①

$\frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}r^2 x$

Area of triangle — ②

$\frac{1}{2}ab\sin\theta = \frac{1}{2}r^2\sin x.$

Area of shaded region.

① - ②

$$\Rightarrow \frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x$$

$$\Rightarrow \frac{1}{2}r^2 (x - \sin x).$$

Area of shaded region $= \frac{5}{100} \times \pi r^2$

$$= 0{\cdot}05\pi r^2$$

$$\therefore \frac{1}{2}r^2 (x - \sin x) = 0\ 05\pi r^2 \quad \times 2$$

$$\therefore \frac{\cancel{r^2}}{\cancel{r^2}} (x - \sin x) = \frac{0{\cdot}1\pi \cancel{r^2}}{\cancel{r^2}}$$

$$\Rightarrow x - \sin x - 0{\cdot}1\pi = 0$$

$$\therefore x - \sin x - \frac{1}{10}\pi = 0$$ as required

b) $$x_{n+1} = x_n - \frac{x_n - \sin x_n - \pi/10}{f'(x)}$$

$$f'(x) = 1 - \cos x$$

$$\Rightarrow x_{n+1} = x_n - \frac{(x_n - \sin x_n - \pi/10)}{1 - \cos x_n}$$

c) $x_0 = 1.2$

Replacing this into the formula we generated above gives;

$x_1 = 1.27245$

$x_2 = 1.26895$

$x_3 = 1.26894\ldots$

$\therefore$ the root $= 1.269$ (3dp)

15 A model for the motion of a small object falling through a thick fluid can be expressed using the differential equation

$$\frac{dv}{dt} = 9.8 - kv,$$

where $v\,\mathrm{m\,s^{-1}}$ is the velocity after t s and k is a positive constant.

(a) Given that $v = 0$ when $t = 0$, solve the differential equation to find v in terms of t and k. [7]

(b) Sketch the graph of v against t. [2]

Experiments show that for large values of t, the velocity tends to $7\,\mathrm{m\,s^{-1}}$.

(c) Find the value of k. [2]

(d) Find the value of t for which $v = 3.5$. [1]

a) $\frac{dv}{dt} = 9\ 8 - kv$

$$\int \frac{dv}{9.8 - kv} = \int dt$$

$$\Rightarrow \int \frac{1}{9.8 - kv}\, dv = \int 1\, dt$$

$$\Rightarrow \frac{1}{-k} \ln(9.8 - kv) = t + c$$

Applying power e on both sides,

$$\ln(9\ 8 - kv) = -k(t + c) \qquad -kt + A$$

→ Another constant.

$$9.8 - kv = e^{(-kt + A)}$$

$$9.8 - kv = e^{-kt} \times e^{A}$$

Another constant

$$\Rightarrow 9.8 - kv = Be^{-kt}$$

when t and v=0, what is the value of our constant?

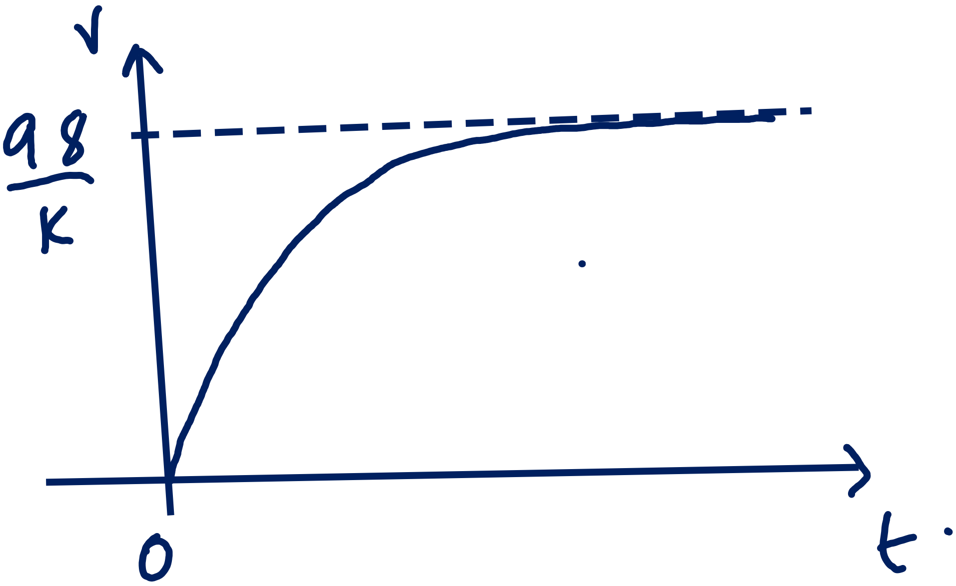
$$9.8 = Be^{0} \quad \therefore \underline{B = 9.8}$$

$$\therefore 9.8 - kv = 9.8e^{-kt}$$

$$9.8 - 9.8e^{-kt} = kv$$

$$\boxed{v = \frac{9.8}{k}\left[1 - e^{-kt}\right]}$$

b)



c) As $t \to \infty$

$v \to 7$.

$\frac{9.8}{k}\left[1 - e^{-\infty}\right] \Rightarrow \frac{9.8}{k}\left[1\right]$

$= \frac{9.8}{k} = 7$.

$k = 1.4$

d) $v = 3.5 \quad t = ?$

$3.5 = \frac{9.8}{1.4}\left[1 - e^{-1.4t}\right]$

$$\Rightarrow \frac{3.5 \times 1.4}{98} = 1 - e^{-1.4t}.$$

$$\frac{1}{2} = 1 - e^{-1.4t}$$

$$\frac{1}{2} = e^{-1.4t}$$

$$\ln\left(\frac{1}{2}\right) = -1.4t$$

$$t = \frac{\ln(1/2)}{-1.4}$$

$$= 0.495 \text{ (3sf)}$$

16 A particle of mass 2 kg slides down a plane inclined at 20° to the horizontal. The particle has an initial velocity of $1.4\,\mathrm{m\,s^{-1}}$ down the plane. Two models for the particle's motion are proposed.

In model A the plane is taken to be smooth. (no friction)

(a) Calculate the time that model A predicts for the particle to slide the first 0.7 m. **[5]**

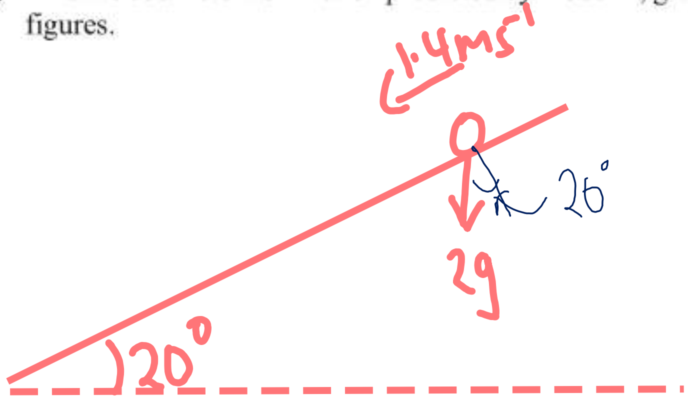
(b) Explain why model A is likely to underestimate the time taken. **[1]**

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

(c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. **[2]**

(d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. **[6]**

a)



Resolving forces: ($\swarrow$)

$2g\sin 20 = \text{Resultant force}$

$F = ma$

$2g\sin 20 = 2a$

$a = g\sin 20 \quad = 3.35\,\mathrm{ms^{-2}}$

$s = 0.7$
$u = 1.4$
$v = x$
$a = 3.35$
$t = ?$

$s = ut + \frac{1}{2}at^2$.

$0.7 = 1.4t + \frac{1}{2}(3.35)t^2 \quad \times 2$

$1.4 = 2.8t + 3.35t^2$

$\Rightarrow 3.35t^2 + 2.8t - 1.4 = 0$

$$\frac{-2.8 \pm \sqrt{(2.8)^2 - 4(-1.4)(3.35)}}{2 \times 3.35}$$

$= 0.352$ or -1.19 Both answers to 3sf

$\rightarrow$ Since time is scalar $t = 0.352$.

b) Friction and air resistance would have slowed down the particle increasing the time taken to travel 0.7m.

(i) $s = 0.7$
$u = 1.4$
$v =$
$a = ?$
$t = 0.8$

$s = ut + \frac{1}{2}at^2$

$0.7 = 1.4(0.8) + \frac{1}{2}a(0.8)^2$

$0.7 - 1.4(0.8) = \frac{1}{2}a(0.8)^2$

$-0.42 = 0.32a.$

$a = -\frac{0.42}{0.32} = \boxed{-1.31\,ms^{-2}}$ (3sf)

$= -\frac{21}{16}$

ii) Resolving forces (↙)

$2g\sin(20) - \underbrace{\mu R}_{\text{Friction} = \mu R} = ma$

Finding R, by resolving (↖)

$2g\cos 20 = R.$

$2g\sin 20 - 2g\cos 20\,\mu = 2(-1.3125)$

$$2g(\sin 20 - \cos 20\,\mu) = 2(-1.3125)$$

$$\sin 20 - \cos 20 \cdot \mu = \frac{-1.3125}{9.8}$$

$$\sin 20 + \frac{1.3125}{9.8} = \cos 20 \cdot \mu$$

$$0.4759\cdot = \cos 20\,\mu$$

$$\mu = \frac{0.4759\cdot}{\cos 20}$$

$$\mu = 0.506 \quad (3sf)$$